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A new method for counting chromatic coefficients

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Abstract

In this paper, Proper-Broken-Cycle Formula is presented. The explicit expression in terms of induced subgraphs for the sixth coefficient of chromatic polynomial of a graph is presented. Also a new proof of Farrell's theorems is given.

1. Introduction

A very interesting relation between zeros of chromatic polynomials and some phenomena in theoretical physics is discovered (see [1,2] with references). It is well known that counting chromatic coefficients (i.e., coefficients of a chromatic polynomial) is important for location of chromatic zeros of graphs.

In this paper we propose a new method for counting chromatic coefficients. This method, called the Proper-Broken-Cycle Formula, is easier than the methods in [3-5]. The Proper-Broken-Cycle Formula can be simply applied for counting the explicit expression for the 4-th, 5-th and 6-th coefficients of the chromatic polynomial of a graph. We consider only simple graphs G with the vertex set $V(G)$ and the edge set $E(G)$. Let $P(G,\lambda)$ be the chromatic polynomial of a graph G. Let $\delta(G)$ be the minimum vertex degree of G. Whitney [5] gave an interpretation of chromatic coefficients of a graph using the concept of a broken cycle. Let $\beta : E(G) \to \{1,2,...,q\}$ be a bijection where q is the size of G. Let C be a cycle of G. Suppose that e is an edge of the cycle C such that $\beta(e) > \beta(x)$ for any other edge x in C. Then the path C-e is called a *broken cycle* in G induced by β.

Theorem 1 (Broken-Cycles Theorem [5]). *Let G be a graph of order n and size q, and let $\beta : E(G) \to \{1,2,...,q\}$ be a bijection. Then*

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$$P(G,\lambda)=\sum_{i=0}^{n-1}(-1)^i h_i \lambda^{n-i}, \tag{1}$$

where h_i is the number of spanning subgraphs of G that have exactly i edges and that contain no broken cycles induced by β .

Let *G* be an edge-labelled graph and let $\{B_1,B_2,\ldots,B_\sigma\}$ be the set of broken cycles of *G*. Let $B_{i_1,i_2,\ldots,i_s}$ be the spanning subgraph of *G* whose edges are precisely those of *G* that belong to at least one of the broken cycles $B_{i_1},B_{i_2},\ldots,B_{i_s}$. Then $B_{i_1,i_2,\ldots,i_s}$ is called *a minimal spanning subgraph of G containing s broken cycles*. Bari and Hall [3] using the principle of inclusion and ecxlusion and the concept of minimal spanning subgraph of *G* containing a number of broken cycles derived the following formula for coefficients of chromatic polynomial of a graph.

Theorem 2 (Bari-Hall Broken-Cycle Formula [3]). *Let G be an edge-labeled graph of order n and size q, and let $\{B_1,B_2,\ldots,B_\sigma\}$ be the set of broken cycles of G induced by the given labeling. Then*

$$P(G,\lambda)=\sum_{i=0}^{n-1}(-1)^i h_i \lambda^{n-i} \tag{2}$$

and

$$h_i=\binom{q}{i}+\sum_{j=2}^{i}\binom{q-j}{i-j}\sum_{s=1}^{\sigma}(-1)^s n_{j,s}, \tag{3}$$

where σ is the number of broken cycles of G, and $n_{j,s}$ is the number of minimal spanning subgraphs of G with j edges and containing s broken cycles.

2. Proper-Broken-Cycle Formula. An application

Using the same method as that of Bari and Hall, we prove (in Theorm 3 below) that the set of *proper broken cycles* of *G* induced by the given labelling can be considered instead of the set of all broken cycles of *G*. Moreover, a family of edge-induced subgraphs can be considered instead of the family of minimal spanning subgraphs. These two facts reduce the number and order of subgraphs studied in the formula for chromatic coefficients of a given graph.

Let $\{B_1,B_2,\ldots,B_\phi\}$ be the set of all proper broken cycles of *G*, where $\phi\le\sigma$ and σ is the numer of all broken cycles of *G*. Let $P_{i_1,i_2,\ldots,i_s}$ be an edge-induced subgraph of *G* generated by the set of all edges of proper broken cycles $B_{i_1},B_{i_2},\ldots,B_{i_s}$, $(i_k\le\phi,\ k=1,\ldots,s)$. So $P_{i_1,i_2,\ldots,i_s}$ contains only the vertices of *G* that belong to an edge of at least one of $B_{i_1},B_{i_2},\ldots,B_{i_s}$. Evidently, each vertex of $P_{i_1,i_2,\ldots,i_s}$ is incident to an edge of a proper broken cycle. For each *s=1,2,...,ϕ,* we

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create the family of $\binom{\phi}{s}$ edge-induced subgraphs of the type $P_{i_1,i_2,\ldots,i_s}$. Each graph $P_{i_1,i_2,\ldots,i_s}$ of the family we call *a minimal subgraph containing s proper broken cycles*. We can formulate the following theorem.

Theorem 3 (Proper-Broken-Cycle Formula). *Let G be an edge-labelled graph of the order n and the size q, and let* $\{B_1,B_2,\ldots,B_\phi\}$ *be the set of all proper broken cycles of G induced by the given labeling. Then*

$$P(G,\lambda)=\sum_{i=0}^{n-1}(-1)^i h_i \lambda^{n-i}, \tag{4}$$

and

$$h_i=\binom{q}{i}+\sum_{j=2}^{i}\binom{q-j}{i-j}\sum_{s=1}^{\phi}(-1)^s g_{j,s}, \tag{5}$$

where ϕ *is the number of proper broken cycles of G, and* $g_{j,s}$ *is the number of minimal subgraphs of G with j edges and containing s proper broken cycles of G.*

Proof: First we introduce some notations. Let ***S*** be the family of all spanning subgraphs with *i* edges of *G*, and let ***B*** be the family of *i*- edge spanning subgraphs of *G* without broken cycles. Notice that ***B*** is created from ***S*** by deleting all elements containing a proper broken cycle. Let ***S'*** be the family of graphs created from ***S*** by deleting all isolated vertices from each graph of ***S***. (Evidently, if *i=0* then ***S'*** consists of the null graph $(\varnothing,\varnothing)$). Let ***B'*** be the subfamily of ***S'*** containing elements without proper broken cycles. Let N_i be the cardinality of ***S*** and $N_i^{'}$ be the cardinality of ***S'***. Notice that $N_i=N_i^{'}=\binom{q}{i}$. From Theorem 1 we have that h_i is the number of spanning subgraphs of *G* with *i* edges that contain no broken cycle. So $h_i=|\boldsymbol{B}|=|\boldsymbol{B'}|$. Let for a subgraph *F* of *G*, b_r be the property that *F* contains the broken cycle B_r, and $\overline{b}_r$ be the property that *F* does not contain B_r, for $r\le\sigma$. Similarly, let p_r be the property that *F* contains the *proper broken cycle* B_r, and $\overline{p}_r$ be the property that *F* does not contain the *proper broken cycle* B_r, where $r\le\phi$. Let $\{f_1,f_2,\ldots,f_t\}$ be a set of properties of a graph. Let $N_i\left(f_1,f_2,\ldots,f\right)_t$ be the number of spanning subgraphs of *G* with *i* edges and with properties $f_1,f_2,\ldots,f_t$. Let $N_i^{'}\left(f_1,f_2,\ldots,f\right)_t$ be the number of elements of ***S'*** with properties $f_1,f_2,\ldots,f_t$. By Theorem 1 we get

$$h_i=N_i\left(\overline{b}_1,\overline{b}_2,\ldots,\overline{b}_\sigma\right)=N_i\left(\overline{b}_1,\overline{b}_2,\ldots,\overline{b}_\phi\right)=N'_i\left(\overline{p}_1,\overline{p}_2,\ldots,\overline{p}_\phi\right). \tag{6}$$

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Therefore by the inclusion - exclusion principle we get

$$\begin{aligned}h_i &= N_i - \left[N_i(b_1) + N_i(b_2) + \cdots + N_i(b_\phi)\right] \\ &+ \left[N_i(b_1,b_2) + N_i(b_1,b_3) + \cdots + N_i(b_{\phi-1},b_\phi)\right] \\ &+ \cdots + (-1)^\phi N_i(b_1,b_2,\ldots,b_\phi) \\ &= N_i^{'} - \left[N_i^{'}(p_1) + N_i^{'}(p_2) + \cdots + N_i^{'}(p_\phi)\right] \\ &+ \left[N_i^{'}(p_1,p_2) + N_i^{'}(p_1,p_3) + \cdots + N_i^{'}(p_{\phi-1},p_\phi)\right] \\ &+ \cdots + (-1)^\phi N_i^{'}(p_1,p_2,\ldots,p_\phi).\end{aligned}$$

The number of *i*-edge elements of the family ***S'*** that contain a given *j*-edge subgraph *F* is $\binom{q-j}{i-j}$. Moreover, if $F = F = P_{i_1,i_2,\ldots,i_s}$ then the term coresponding to *F* appears with the coefficient $(-1)^s$ in the above formula. Therefore, if there are $g_{j,s}$ subgraphs of *G* with *j* edges and the edge induced by *s* proper broken cycles, then they contribute $(-1)^s g_{j,s}\binom{q-j}{i-j}$ to h_i. The proof is done.

□

In this paper we use Theorem 3 instead of Theorem 2. Let t_1 be the number of subgraphs isomorphic to K_3 in *G*. Let us assume that $T_i, i = 2,\ldots,31$ and $F_j, j = 0,1,\ldots 11$ be graphs presented in Figures 1, 2 and 3.

Let t_i, $i = 2,\ldots,31$ (resp. $f_j, j = 0,1,\ldots 11$), be the number of induced subgraphs of *G* isomorphic to T_i (resp.F_i). Farrell [4] proved the following two significant theorems concerning two coefficients of the chromatic polynomial of a graph.

Theorem 4[4]. *The coefficient of* λ^{n-3} *in P(G,λ) is* $-\binom{q}{3} + (q-2)t_1 + t_2 - 2t_3$.

Theorem 5[4]. *The coefficient of* λ^{n-4} *in P(G,λ) is*

$\binom{q}{4} - \binom{q-2}{2} t_1 + \binom{t_1}{2} - (q-3) t_2 - (2q-9) t_3 - t_4 + t_5 + 2t_6 + 3t_7 - 6t_8$.

Now we apply the Proper Broken-Cycle Formula presented in Theorem 3 to prove the above theorems in different and simpler way in comparison with the proof given in [4]. First we study properties of subgraphs of a graph *G,* involved in h_i, for *i=2,3,4,….*

Let for a graph *G,* the numbers ϕ and $g_{j,s}$ be defined as in Theorem 3, and let $w_j(G) = \sum_{s=1}^{\phi}(-1)^s g_{j,s}$. The number $w_j(G)$ we call the *j-weight* of the graph G.

Proposition 6. Let F be an induced subgraph of G such that $\delta(F) = 1$. Then any spanning subgraph of F does not give its contribution to $w_j(G)$, for $j = 2,3\ldots$.

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Proof: Let F be an induced subgraph of a graph G such that $\delta(F)=1$. Let H be a spanning subgraph of F. Suppose that H is counted in j-weight of G. Then H has j edges and it is covered by a set of proper broken cycles. If H contains an edge incident to a 1-degree vertex in F, then this edge cannot belong to any broken cycle, a contradiction. Therefore H contains an isolated vertex, a contradiction (since, by definition of $g_{j,s}$, each vertex should be covered by an edge of a proper broken cycle). The proof is done.

□

Proposition 7. *Let j be the integer,* $2 \le j$ *, and let F be an induced subgraph of G. If* $|V(F)| > j + \lfloor \frac{j}{2} \rfloor$ *then F is not covered by any set of proper broken cycles with j as the sum of their edges. Moreover, if* $|E(F)| \le j$ *then* $w_j(F) = 0,\ j = 2,3,\ldots$.

Proof: Note that F contains a vertex of degree at most 1. Thus the result follows immediately from Proposition 6 and Theorem 3.

□

Proposition 8. *Let* $2 \le j$ *. Then* $w_j(G) = \sum_F w_j(F)$*, where the sum is taken over all induced subgraphs F of G with the minimum degree at least 2 and such that* $|V(F)| \le j + \lfloor \frac{j}{2} \rfloor$ *and* $|E(F)| > j$ *.*

Proof: Immediately from the definition of the broken cycle and from Propositions 6, 7.

□

Immediately from Proposition 8 we have the following result.

Corollary 9. *For a graph G we get*

$w_2(G) = w_2(T_1)\cdot t_1$,

$w_3(G) = w_3(T_2)\cdot t_2 + w_3(F_0)\cdot f_0 + w_3(T_3)\cdot t_3$,

$w_4(G) = \sum_F w_4(F)$, *where the sum is taken over all induced subgraphs F of G of the order 4,5,6 with* $\delta(F) > 1$,

$w_5(G) = \sum_F w_5(F)$, *where the sum is taken over all induced subgraphs F of G of the order 4,5,6,7 with* $\delta(F) > 1$.

Proposition 10. *All weights are independent of an edge labelling of a graph.*

Proof: Immediately from the Proper Broken-Cycle Formula.

□

Evidently, by Corollary 9, the *2*-weights of G equals $-t_1$. Similarly *3*-weights can be easily obtained. Notice that if a graph has at most 3 proper broken cycles

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the *4-,* and *5*-weights can be easily obtained as well. The *4-,* and *5*-weights for other graphs we have been found by means of the computer computation. Table 1 contains the *3*-,*4-,* and *5*-weights for the graphs of the orders 4 and 5 with the minimum degree at least 2. Table 2 contains the *5*-weights for the graphs of order 6 with the minimum degree at least 2.

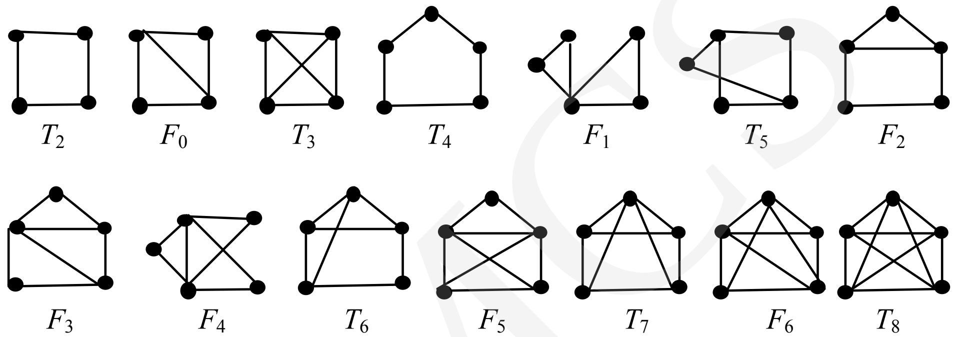

Fig. 1. The list of graphs of the orders 4 and 5 with minimum degree at least 2

In order to count h_i, for $i=4,5$, we use some special functions, defined below. Let *D, F, H* be the graphs of the orders 4, 5, 6, respectively. Let F_0, F_1, F_2, F_5 be the graphs presented in Fig. 1, and let F_7, F_8 be the graphs presented in Fig.3. We define two functions *p, s* for the graphs *D* and *F,* respectively as follows:

p(D) = the number of spanning subgraphs of *D* isomorphic to F_0,

s(F) = the number of spanning subgraphs of *F* isomorphic to F_1.

Moreover, we define two functions *u, z* for *F* and two functions *x, y* for *H* as follows:

u(F) = the number of spanning subgraphs of *F* isomorphic to F_2, and such that the 4-cycles do not contain any chord in *G*,

z(F) = the number of spanning subgraphs of *F* isomorphic to F_5,

x(H) = the number of spanning subgraphs of *H* isomorphic to F_7, and such that the 4-cycles do not contain any chord in *G*,

y(H) = the number of spanning subgraphs of *H* isomorphic to F_8.

The values of functions *p, s, u, z* are given in Table 1, and the values of functions *x* and *y* are given in Table 2, where *X=x(H)* and *Y=y(H).*

Proof: *(of Theorems 4 and 5)* We count h_3, h_4 from Theorem 3 and Corollary 9. Evidently $w_2(G) = -t_1$ and $w_3(G) = -t_2 + 2t_3$. Let $d_4 = \sum_H w_4(H)$, where the sum is taken over all the induced subgraphs *H* of the order 6 in *G* and such that $\delta(H) > 1$. Note that the contribution of a graph *H* of the order 6 into *4*-weight equals the number of spanning subgraphs of *H* isomorphic to $2K_3$. Therefore, from Table 1, we have

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$$w_4(G) = d_4 + f_0 + 3t_3 - t_4 + f_1 + t_5 + f_3 + 2t_6 + 2f_5 + 5t_7 + 6f_6 + 9t_8.$$

Evidently, $w_4(F_0) = 1 = w_4(F_1) = w_4(2K_3)$ and $d_4 = \binom{t_1}{2} - \sum_D p(D) - \sum_F s(F)$, where the sums are taken over all induced subgraphs D of the order 4 and all induced subgraphs F of the order 5 in G and such that $\delta(D) > 1$ and $\delta(F) > 1$. Hence, $w_4(G) = \binom{t_1}{2} - 3t_3 - t_4 + t_5 + 2t_6 + 3t_7 - 6t_8$.

According to the Proper Broken-Cycle Formula presented in Theorem 3, we obtain h_3, h_4.

Table 1. Graphs of the orders 4 and 5 with the minimum degree at least 2 and their *j*-weights, *j=3,4,5,* as well as their special functions

$F=$	T_2	F_0	T_3	T_4	F_1	T_5	F_2	F_3	F_4	T_6	F_5	T_7	F_6	T_8
$w_3(F)$	-1	0	2	0	0	0	0	0	0	0	0	0	0	0
$w_4(F)$	0	1	3	-1	1	1	0	1	0	2	2	5	6	9
$w_5(F)$	0	0	-2	0	0	1	1	0	0	2	-2	-2	-12	-36
$p(F)$	0	1	6											
$s(F)$				0	1	0	0	1	0	0	2	2	6	15
$u(F)$				0	0	0	1	0	0	2	0	4	0	0
$z(F)$				0	0	0	0	0	0	0	1	0	6	30

□

The following theorem presents the sixth chromatic coefficient for a graph *G*.

Theorem 11. *The coefficient of* λ^{n-5} *in* $P(G,\lambda)$ *is* $(-1)h_5$*, where*

$$h_5 = \binom{q}{5} - \binom{q-2}{3}t_1 + (q-4)\binom{t_1}{2} - \binom{q-3}{2}t_2 + (t_2 - 2t_3)t_1 + (q^2 - 10q + 30)t_3 - t_4$$
$$+ (q-3)t_5 + 2(q-5)t_6 + 3(q-6)t_7 - 6(q-8)t_8 - t_9 + t_{10} + 2t_{11} + 2t_{12}$$
$$+ t_{13} - t_{14} + t_{15} + 3t_{16} + 4t_{17} + 4t_{18} - 2t_{19} + 4t_{20} + t_{21} - 4t_{22} - 3t_{23} - 4t_{24}$$
$$- 5t_{25} - 4t_{26} - 6t_{27} - 8t_{28} - 16t_{29} - 12t_{30} + 24t_{31},$$

Where t_1 *is the number of* K_3*in* G *and* t_i, $i = 2,...,28$, *are the numbers of subgraphs of* G *isomorphic to graphs* T_i *presented in Figures 1-2.*

Proof: Using Theorem 3 and Corollary 9 we count h_5. Let $d_5 = \sum_F w_5(F)$, where the sum is taken over all induced subgraphs F of the order 7 in G such that $\delta(F) > 1$. Let F_9, F_{10}, F_{11} be the graphs presented in Fig. 3. Notice that $w_5(F_{10}) = 0$, $w_5(F_9) = 1$, $w_5(F_{11}) = -2$.

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Therefore

$$d_5 = t_1 \cdot t_2 - 2 \cdot t_1 \cdot t_3 + 2 \cdot 4 \cdot t_3 - \sum_F \left(u(F) - 2z(F)\right) + \sum_H \left(2y(H) - x(H)\right),$$

where the sums are taken over all induced subgraphs F of the order 5 and all induced subgraphs H of the order 6 in G, with $\delta(F) > 1$ and $\delta(H) > 1$.

From Proposition 8 and Corollary 9, we have

$$w_5(G) = d_5 + t_3 \cdot w_5(T_3) + \sum_F w_5(F) + \sum_H w_5(H),$$

where the sums are taken over all induced subgraphs F of the order 5 and all induced subgraphs H of the order 6 in G, with $\delta(F) > 1$ and $\delta(H) > 1$. Hence, from Theorem 3, we get

$$h_5 = \binom{q}{5} - \binom{q-2}{3} \cdot t_1 + \binom{q-3}{2}\left(-t_2 + 2t_3\right) + (q-4)\left(\binom{t_1}{2} - 3t_3 - t_4 + t_5 + 2t_6 + 3t_7 - 6t_8\right)$$
$$+ t_1 \cdot t_2 - 2t_1 \cdot t_3 + 6t_3 + t_5 - 6t_7 + 24t_8 - t_9 + t_{10} + 2t_{11} + 2t_{12} + t_{13} - t_{14} + t_{15}$$
$$+ 3t_{16} + 4t_{17} + 4t_{18} - 2t_{19} + 4t_{20} + t_{21} - 4t_{22} - 3t_{23} - 4t_{24} - 5t_{25} - 4t_{26} - 6t_{27}$$
$$- 8t_{28} - 16t_{29} - 12t_{30} + 24t_{31}.$$

Finally, from the simple algebraic reduction, we get the result.

□

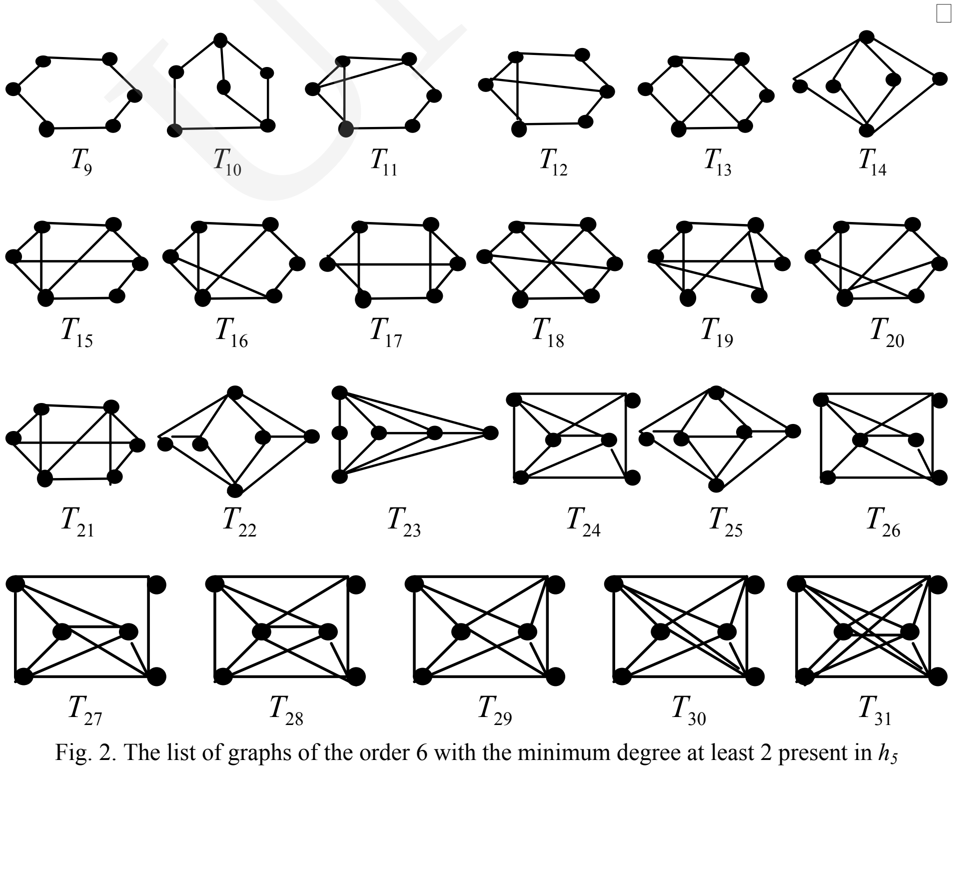

Fig. 2. The list of graphs of the order 6 with the minimum degree at least 2 present in h_5

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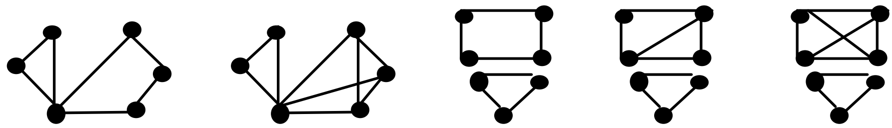

(0,0,1) F_7 (0,0,-2) F_8 (0,0,1) F_9 (0,0,0) F_{10} (0,0,-2) F_{11}

Fig. 3. Other graphs of the minimum degree *2* and the order *6* or *7* with their 3-, *4*- and *5*-weights

The following Table 2 contains the adjacency matrices of graphs *H* of the order 6 with the minimum degree at least 2 and their *5*-weights (**bold,** in the right upper corner**)**, where X=*x(H)*, Y=y(*H*).

Table 2. The adjacency matrices of graphs *H* of the order 6 with the minimum degree at least 2 and their *5*-weights (**bold,** in the right upper corner), where X=*x(H)*, Y=y(*H*)

T_{31} : **-96** 011111 101111 110111 111011 111101 111110 X=0 Y=60	**-48** 001111 001111 110111 111011 111101 111110 X=0 Y=24	**18** 000111 001111 010111 111011 111101 111110 X=0 Y=9	T_{30} : **-24** 001111 001111 110011 110011 111101 111110 X=4 Y=8	T_{29} : **-16** 001111 001111 110011 110011 111100 111100 X=0 Y=0	**0** 000111 000111 000111 111011 111101 111110 X=0 Y=0	T_{28} : **-6** 000111 001111 010111 111001 111001 111110 X=6 Y=2	**-4** 000011 001111 010111 011011 111101 111110 X=0 Y=2
-8 000111 001011 010111 101011 111101 111110 X=0 Y=4	T_{27} : **3** 000011 001111 010111 011011 111100 111100 X=9 Y=0	**-2** 000011 001111 010111 011001 111001 111110 X=0 Y=1	**0** 000011 000111 000111 011011 111101 111110 X=0 Y=0	T_{26} : **2** 000111 000111 000111 111001 111001 111110 X=6 Y=0	**-8** 000111 001011 010011 100011 111101 111110 X=0 Y=4	**0** 000111 001011 010111 101001 111001 111110 X=2 Y=1	T_{25} : **-3** 000111 001011 010111 101011 111100 111100 X=2 Y=0
T_{24} : **4** 000111 001111 010111 111000 111001 111010 X=8 Y=0	**4** 000011 001101 010111 011011 101100 111100 X=4 Y=0	**-2** 000101 001011 010011 100011 011101 111110 X=0 Y=1	**0** 000011 001101 010110 011011 101101 110110 X=0 Y=0	**0** 000011 000101 000111 011011 101101 111110 X=0 Y=0	**1** 000011 000111 000111 011001 111001 111110 X=1 Y=0	T_{23} : **1** 000011 000111 000111 011011 111100 111100 X=4 Y=0	**0** 000011 000011 000111 001011 111101 111110 X=0 Y=0

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0 000011 001100 010111 011011 101101 101110 X=0 Y=0	**6** 000111 000111 000111 111000 111001 111010 X=6 Y=0	T_{22} : **-4** 000111 001011 010011 100011 111100 111100 X=0 Y=0	T_{21} : **3** 001111 000111 100011 110001 111000 111100 X=2 Y=0	T_{20} : **4** 001101 000111 100011 110001 011001 111110 X=0 Y=0	**0** 000011 000011 000011 000011 111101 111110 X=0 Y=0	T_{19} : **0** 000011 000011 000111 001011 111100 111100 X=2 Y=0	**0** 000011 000011 000110 001011 111101 110110 X=0 Y=0
0 000011 000101 000110 011010 101101 110110 X=0 Y=0	**0** 000011 000110 000111 011010 111101 101010 X=0 Y=0	**1** 000011 000110 000111 011001 111001 101110 X=1 Y=0	**2** 000011 000111 000111 011000 111001 111010 X=2 Y=0	T_{18} : **4** 000111 000111 000111 111000 111000 111000 X=0 Y=0	T_{17} : **4** 001110 000111 100011 110001 111000 011100 X=0 Y=0	F_8 : **-2** 001001 000111 100001 010011 010101 111110 X=0 Y=1	**2** 001010 000111 100001 010011 110101 011110 X=2 Y=0
0 001010 000101 100011 010011 101101 011110 X=0 Y=0	**0** 001010 000111 100011 010011 111100 011100 X=0 Y=0	T_{16} : **4** 001100 000111 100011 110001 011001 011110 X=1 Y=0	T_{15} : **3** 000011 000111 000111 011001 111000 111100 X=2 Y=0	T_{14} : **-1** 011101 100010 100010 100010 011101 100010 X=0 Y=0	**0** 011000 101000 110011 000011 001101 001110 X=0 Y=0	**0** 010010 100010 000011 000011 111101 001110 X=0 Y=0	**1** 011000 101011 110100 001011 010100 010100 X=1 Y=0
0 000011 000011 000110 001001 111001 110110 X=0 Y=0	T_{13} : **1** 001100 000011 100011 100011 011100 011100 X=0 Y=0	T_{12} : **2** 000101 000011 000111 101010 011100 111000 X=0 Y=0	T_{11} : **2** 011000 100001 100110 001011 001101 010110 X=0 Y=0	**0** 011000 101001 110100 001011 000101 010110 X=0 Y=0	**0** 011001 100001 100100 001011 000101 110110 X=0 Y=0	**1** 011000 100001 100101 001011 000101 011110 X=1 Y=0	**0** 011000 100001 100101 001010 000101 011010 X=0 Y=0
0 011100 101000 110000 100011 000101 000110 X=0 Y=0	T_{10} : **1** 011001 100100 100010 010010 001101 100010 X=0 Y=0	F_7 : **1** 011000 101000 110101 001010 000101 001010 X=1 Y=0	**0** 011000 100001 100100 001011 000101 010110 X=0 Y=0	T_9 : **-1** 011000 100001 100100 001010 000101 010010 X=0 Y=0			

3. Final remarks

The values of h_5, h_4 can be applied to study sufficient conditions for the existence of complex roots of chromatic polynomials. The conditions in terms h_2, h_3 are presented in [6] and [7], respectively. The new Proper-Broken-Cycle

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Formula presented in Theorem 3 can be simply applied for counting explicit expression for the next coefficients of chromatic polynomial of a graph.

References

[1] Sokal A.D., *Bounds on the Complex Zeros of (di)chromatic Polynomials and Potts-model Partition Functions*, Combin., Probab. Comput., 10 (2001) 41.
[2] Sokal A.D., *Chromatic Roots are Dense in the Whole Complex Plane*, Combin., Probab. Comput., 13(2) (2004) 221.
[3] Bari R.A., Hall D.W., *Chromatic Polynomials and Whitney's broken circuits*, Journal of Graph Theory, 1 (1977) 269.
[4] Farrell E.J., *On chromatic coefficients*, Discrete Mathematics, 29 (1980) 257.
[5] Whitney H., *A logical expansion in mathematics*, Bull. Amer. Math. Soc., 38 (1932) 572.
[6] Brown J., *On roots of chromatic polynomials*, J. Combin. Theory S.B., 72 (1998) 251.
[7] Bielak H., *Roots of chromatic polynomials*, Discrete Mathematics, 231 (2001) 97.

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